B.A/ B.Sc	Semester-III	Credits:4
Course:3	ABSTRACT ALGEBRA	Hrs/Weak:5

Course Outcomes:

After successful completion of this course, the student will be able to;

- acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
- get the significance of the notation of a normal subgroups.
- get the behavior of permutations and operations on them.
- study the homomorphisms and isomorphisms with applications.
- Understand the ring theory concepts with the help of knowledge in group theory and to prove the theorems.
- Understand the applications of ring theory in various fields.

UNIT I: **(12 Hours)**

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II: **(12 Hours)**

SUBGROUP: Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroups. Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. **Co-sets and Lagrange's Theorem:** Cosets Definition – properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT III: **(12 Hours)**

NORMAL SUBGROUPS: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group.

UNIT IV: **(12 Hours)**

HOMOMORPHISM : Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

PERMUTATIONS: Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

UNIT V: **(12 Hours)**

RINGSDefinition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings, Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK :

1. A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published by S.Chand & Company, New Delhi.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-III, ABSTRACT ALGEBRA

Unit	**TOPIC**	**S.A.Q**(including choice)	**E.Q**(including choice)	**Total Marks**
I	Groups	2	2	30
II	Subgroups, Cosets & Lagrange's theorem	1	2	25
III	Normal Subgroups	1	2	25
IV	Homomorphism and Permutations	2	2	30
V	Rings	2	2	30
Total		8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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Total Marks = 75 M

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MODEL QUESTION PAPER (Sem-End)
B.A./B.Sc. DEGREE EXAMINATIONS

Semester - III

Course-3: ABSTRACT ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION – A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Show that the set G=$\{x/x = 2^a 3^b \text{ and } a, b \in Z\}$ is a group under multiplication
2. Define order of an element. In a group G, prove that if $a \in G$ then $O(a) = O(a)^{-1}$.
3. If H and K are two subgroups of a group G, then prove that HK is a subgroup ⇔ HK=KH
4. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.
5. Examine whether the following permutations are even or odd

   i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$ ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$
6. If f is a homomorphism of a group G into a group G', then prove that the kernel of f is a normal of G.
7. Prove that the characteristic of an integral domain is either prime or zero.
8. Define a Boolean Ring and Prove that the Characteristic of a Boolean Ring is 2.

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. a) Show that the set of nth roots of unity forms an abelian group under multiplication.

   (Or)

   b) In a group G, for $a, b \in G$, O(a)=5, b ≠ e and $aba^{-1} = b^2$. Find O(b).

10. a) The Union of two subgroups is also a subgroup □ one is contained in the other.

    (Or)

    b) State and prove Langrage's theorem.

11. a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

    (Or)

    b) Define Normal Subgroup. Prove that a subgroup H of a group G is normal iff $xHx^{-1} = H \ \forall \ x \in G$.

12. a) State and prove fundamental theorem of homomorphisms of groups.

    (Or)

    b) Let S_n be the symmetric group on n symbols and let A_n be the group of even permutations. Then show that A_n is normal in S_n and O(A_n) =$\frac{1}{2}(n!)$

13. a) Prove that every finite integral domain is a field.

    (Or)

    b) Let S be a non empty sub set of a ring R. Then prove that S is a sub ring of R if and only if a-b ∈ S and ab ∈ S for all a, b ∈ S.